Mathematics education research and policy groups in the United States have advocated for mathematics teaching and curriculum standards for decades. The *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 1989) contains mathematics content goals for the different grade-bands as well as goals that emphasize practices such as communication, problem solving, reasoning, and making mathematical connections. The *Principles and Standards for School Mathematics* (NCTM, 2000) includes five Process Standards (problem solving, reasoning and proof, communication, connections, and representation) that reflect a similar trend toward creating mathematics teaching and learning environments that emphasize both learning mathematics content as well as developing beliefs and dispositions as a mathematical learner. Along similar lines, the National Research Council (NRC, 2001) outlines five strands of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) that together support successful mathematics learning. A common theme in these prior standards initiatives is a dual focus on learning both mathematics content and mathematical practices.

The Common Core State Standards (CCSS) build on these prior efforts (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010g). Whereas the ideas are not new (Koestler, Felton, Bieda, & Otten, 2013; Reys, 2013), the fact that these CCSS have been adopted by most states in the country and are becoming the basis for mathematics instruction and assessment across the United States distinguishes this initiative from prior efforts and makes it imperative that we as mathematics educators pay attention to the consequences of this widespread adoption and implementation.
In the introduction of the CCSS for Mathematics (CCSSM; NGA & CCSSO, 2010b), we read:

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. (p. 4)

In this book, we address the gap reflected in the statement, “it is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners” by providing a collection of pedagogical practices that are intended to support ELLs with the content and language demands of the CCSSM. In particular, the authors of the different chapters have all chosen to address at least one of the Standards for Mathematical Practice (MP) in the CCSSM. These MPs are closely related to some of the prior efforts mentioned above, in particular, the Process Standards (NCTM, 2000; see Koestler et al., 2013, for an in-depth look at the connections between these two sets of standards) and the strands of mathematical proficiency (NRC, 2001). Whereas the CCSSM content standards can also create language and content demands for ELLs (and the chapters in this book illustrate some of these demands), the MPs call for the engagement of students in such practices as “construct viable arguments and critique the reasoning of others,” “model with mathematics,” and “attend to precision,” all practices that involve mathematical communication and could be perceived as challenging when working with ELLs. It is important to point out what Moschkovich (1999b) wrote in reference to the increased emphasis on mathematical communication included in prior calls for change (e.g., NCTM, 1989). She argued:

On the one hand, the increased expectation for Latino students who are English-language learners to participate in public conversations might increase the possibilities that these students will be assessed as deficient in mathematics because of their developing oral language skills. . . . On the other hand, this change might also provide more opportunities for English language learners to participate in purposeful and contextual conversations with other speakers, creating an environment that can support both language and conceptual development. (p. 7)

The editors and authors of this book take this message seriously, as we view the current MPs as opportunities for ELLs to be part of learning and teaching environments that reflect the different expertise and backgrounds (including languages) that all students bring to the classroom. Hence, in this book we emphasize the mathematical practices (MPs). Each chapter highlights, via detailed classroom-based vignettes, specific pedagogical practices that teachers can use to support ELLs with these MPs. The chapters also reflect the principles for teaching mathematics to ELLs that are outlined in the CCSS document, “Application of Common Core State Standards for English Language Learners” (NGA & CCSSO, 2010a), as well as those outlined in other documents, as we indicate below. In particular, the chapters in this book highlight the following practices and principles for effective mathematics instruction with ELLs:

- ELLs need to have opportunities to communicate mathematically in the classroom (orally and in writing). This entails more than learning vocabulary. Teachers should engage
ELLs in mathematical discourse by encouraging them to make conjectures and explain their thinking. ELLs should have opportunities to read and discuss mathematical texts (Moschkovich, 2013a; NCTM, 2013). (See Chapters 2, 3, 5, 6, 7, and 10.)

• Teachers should use (and encourage students to use) multiple resources to support sense making and communication (e.g., objects, drawings, graphs, gestures, and home languages). Teachers should draw upon these multiple resources as they launch and discuss tasks, and students should be encouraged to draw on these resources as tools for sense making (Coggins, Kravin, Coates, & Carroll, 2007; Moschkovich, 2013a; Ramirez & Celedón-Pattichis, 2012). (See Chapters 4, 6, 8, and 10.)

• Teachers should build on the cultural and linguistic differences that students bring to the classroom. “[These] should be viewed as intellectual resources rather than as deficits and should be used in the classroom to connect to prior knowledge” (Ramirez & Celedón-Pattichis, 2012, p. 21). For example, teachers can connect to students’ home languages, their experiences with mathematical ideas and practices in home and community settings, and other out-of-school experiences to support students’ understanding of mathematical concepts (Kersaint, Thompson, & Petkova, 2013). (See Chapters 2, 7, 8, and 9.)

• ELLs should work on challenging tasks that reflect their mathematical potential. The level of the mathematical tasks should not be lowered based on language considerations. Tasks should include opportunities for students to reason about mathematical ideas; to pose, model, and solve problems; to analyze patterns; and to communicate and justify their thinking. Teachers need to provide supports so that ELLs can access these tasks (Kersaint et al., 2013; Ramirez & Celedón-Pattichis, 2012). (See Chapters 4, 5, and 9.)

• ELLs benefit from scaffolding strategies to support making sense of, solving, and communicating about mathematical tasks. These strategies include providing a range of opportunities for student-to-student interaction (in pairs and in small groups), selecting problems with multiple points of entry, using visuals and concrete materials, and strategic pairing of English learners and bilingual students (Carr et al., 2009a; Coggins et al., 2007). (See Chapters 3, 4, and 9.)

In the next section, we provide a brief overview of each chapter, followed by suggestions about ways to use this book in varied professional development and teacher preparation settings.

In Chapter 2, Moschkovich focuses on the pedagogical practice of building on student language resources during mathematical discussions and suggests how teachers can build on these resources in ways that help ELLs meet the demands of three of the CCSSM MPs: MP #2 (reason abstractly and quantitatively), MP #3 (construct viable arguments and critique the reasoning of others), and MP #6 (attend to precision). She uses detailed mathematical discussion episodes involving upper elementary and middle school students working on geometry tasks to demonstrate various resources that ELLs draw upon as they communicate their mathematical reasoning. Additionally, the chapter outlines ways that teachers can build on these resources to support students with the CCSSM MPs.
In Chapter 3, Pinnow and Chval use four excerpts from a third-grade mathematics lesson on multiplication and division to demonstrate the pedagogical practice of organizing lessons and scaffolding academic language to support ELLs with MP #1 (make sense of problems and persevere in solving them). The lesson excerpts include examples of how the teacher selected and introduced problem solving–based tasks to students, and ways that she emphasized mathematical communication and positioned ELLs to be successful and persistent problem solvers.

In Chapter 4, Celedón-Pattichis and Musanti use episodes from two kindergarten classrooms to further explore MP #1. They demonstrate how kindergarten teachers use the pedagogical practice of PRO-Literacy, a set of instructional strategies that focus on developing academic literacy through problem solving, to support emergent bilingual kindergarteners as the students make sense of and persevere in solving word problems. The authors describe how the two kindergarten teachers, one in a dual-language (English/Spanish) classroom and the other in a setting where English is used for all instruction, consider the content and linguistic demands of word problems and then support students as the students listen to and retell problems, use various tools to represent and solve the problems, and communicate their reasoning.

In Chapter 5, Zahner and Willey focus on two pedagogical practices that teachers can use to meet the needs of ELLs in a CCSSM-aligned classroom: selecting tasks that facilitate the use of academic language and then seizing opportunities to build on students’ contributions of academic language during instruction. Using detailed examples from middle school lessons on rational numbers and algebraic reasoning, the authors illustrate these pedagogical practices in action, with particular attention on how they facilitate two CCSSM MPs that are particularly salient for ELLs, MP #3 (construct viable arguments and critique the reasoning of others) and MP #6 (attend to precision).

In Chapter 6, Fernandes, Civil, and Kahn demonstrate how the pedagogical practice of broadening mathematical communication, which involves attending to students’ gestures, drawing, and actions with objects, in addition to their oral and written language, can support ELLs’ success with the CCSSM. The authors use excerpts of problem solving–based interviews with middle school students to illustrate different applications of this pedagogical practice. The examples in this chapter show how teachers can consider multiple modes of communication to enhance ELLs’ participation in mathematics discussion and, in particular, their engagement with MP #3 (construct viable arguments and critique the reasoning of others), MP #6 (attend to precision), and MP #7 (look for and make use of structure).

In Chapter 7, Thompson and Radosavljevic focus on the pedagogical practice of Socratic Seminars, which has great potential for supporting ELLs with the increased emphasis on literacy-based mathematical activity in the CCSSM. During a Socratic Seminar, students read a mathematics-rich expository text and then engage with their classmates in discourse about that text. The chapter illustrates how Socratic Seminars can support ELLs with MP #3 (construct viable arguments and critique the reasoning of others) and MP #6 (attend to precision), and provides resources and practical guidance for teachers wishing to implement this practice in their classrooms.

In Chapter 8, LópezLeiva introduces the pedagogical practice of extended related activity (ERA) as a way to facilitate ELLs’ participation in MP #4 (model with mathematics). In an ERA, students
reenact an authentic community practice to help them explore and reason about the mathematical content of the practice. LópezLeiva draws on episodes from a bilingual afterschool mathematics club for elementary-grade students to illustrate the practice of ERA, and to describe how the practice can support ELLs’ participation in mathematical modeling. The chapter also includes suggestions for how teachers can adapt the practice for use in classroom (versus afterschool) settings.

In Chapter 9, Anhalt also focuses on MP #4 and describes how teachers can use two different pedagogical practices—analytic scaffolding and social scaffolding—to help ELLs meet the linguistic and content demands of mathematical modeling tasks. Anhalt uses a fourth-grade modeling task that addresses operations and algebraic thinking and a middle school modeling task focused on patterns and functional relationships to illustrate how teachers can implement the two scaffolding practices in ways that support ELLs with MP #4.

In Chapter 10, Kitchen closes the book with a focus on pedagogical practices related to assessment. Specifically, Kitchen introduces the discursive assessment protocol (DAP), a culturally relevant formative assessment tool that teachers can use to promote mathematical sense making with ELLs, as well as their participation in various mathematical practices (MP #1–#5). Kitchen uses detailed examples of students’ solutions and explanations for problem solving tasks involving fractions to demonstrate how the DAP assessment protocol can also support ELLs’ success with Grade 5 CCSSM content standards.

This book is intended to be used in professional development or teacher education settings with practicing and prospective mathematics teachers. More specifically, teachers might use this book to support understanding, action, and reflection related to implementing the MPs with ELL students. While the chapters focus on different grades (spanning Grades K–8) and different MPs, they all share a similar structure. This common structure links research and practice by first discussing the content and linguistic demands of a specific aspect of CCSSM and then introducing a research-based pedagogical practice that constitutes the core of the chapter. The pedagogical practices are each illustrated via detailed classroom vignettes that include excerpts of classroom discussions, sample mathematical tasks, lists of suggested teacher actions and resources, among other details. Each chapter concludes with questions for reflection and suggestions for action plans. The suggested action plans could lead to small groups of teachers trying out ideas in their classrooms and then reporting back on their experiences to their group. Individual teachers reading this book may want to try some of the pedagogical practices discussed in the various chapters. The reflection questions can serve as prompts for group discussions or individual reflections when using this book in teacher education courses or professional development.